

# Holographic Renormalization of New Massive Gravity

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## Abstract

We study holographic renormalization for three dimensional new massive gravity (NMG). By studying the general fall off conditions for the metric allowed by the model at infinity, we show that at the critical point where the central charges of the dual CFT are zero it contains a leading logarithmic behavior. In the context of AdS/CFT correspondence it can be identified as a source for an irrelevant operator in the dual CFT. The presence of the logarithmic fall off may be interpreted as the fact that the dual CFT would be a LCFT.

# 1 Introduction

Although Einstein gravity in three dimensions even with cosmological constant has no propagating modes, adding higher derivative terms to the action given raise to non-trivial propagating degrees of freedom. The most well-known three dimensional gravity with higher derivative terms is topologically massive gravity (TMG) [1, 2] whose action is given by the Einstein-Hilbert action plus three dimensional gravitational Chern-Simons term. This model has propagating massive graviton. The model admits several vacua including  $AdS_3$  solution.

It is generically expected that adding higher derivative terms to the action leads to an instability due to the present of ghost-like modes. Nevertheless it was shown [3] that TMG model is stable above the  $AdS_3$  vacuum at a critical value of the coefficient of the Chern-Simons term where the model would be chiral in the sense that all the left moving excitations of the theory are pure gauge. Therefore we are only left with the right moving excitations<sup>1</sup>.

Soon after it was shown in [5] that the linearized equations of motion at the critical point have a solution which may be interpreted as a left moving excitation<sup>2</sup>. However it is worth mentioning that this solution which has the same asymptotic behavior as AdS wave solution [14, 15] does not obey Brown-Henneaux boundary conditions [16]. Therefore if we restrict ourselves to solutions which satisfy the Brown-Henneaux conditions one may still have chiral theory. On the other hand if we relax the boundary conditions the theory will not be chiral and indeed it was conjectured in [5] that the dual theory (in the sense of AdS/CFT correspondence [17]) could be a logarithmic CFT (LCFT) [18].

Holographic renormalization of TMG model has been studied in [19] where the authors have also computed one and two point functions of the dual CFT. It was shown that the two point functions of the left mover sector at the critical point are those of a LCFT with zero central charge (See also [20]).

Another three dimensional gravity with higher derivative terms has been introduced in [21]. The corresponding action is given by

$$S = \frac{1}{16\pi G} \int_R d^3x \sqrt{-G} [R - 2\lambda] - \frac{1}{m^2} \frac{1}{16\pi G} \int_R d^3x \sqrt{-G} \left[ R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right]. \quad (1.1)$$

This model, known as new massive gravity (NMG), admits several vacua including  $AdS_3$  vacuum. It is believed that NMG model on an asymptotically locally  $AdS_3$  geometry may have a dual CFT whose central charges are given by [22, 23] (see also [24, 25])

$$c_L = c_R = \frac{3l}{2G} \left( 1 - \frac{1}{2m^2 l^2} \right). \quad (1.2)$$

At the critical value,  $m^2 l^2 = \frac{1}{2}$ , where the central charges are zero it has been shown [26] that the model admit a new vacuum solution which is not asymptotically locally  $AdS_3$ . More precisely at the critical point one finds AdS wave solution whose metric is given by [26]

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} \left( -2dudv - F(\rho)du^2 \right), \quad (1.3)$$

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<sup>1</sup>For a rigorous definition of chiral CFT see [4].

<sup>2</sup>Whether at the critical point the model is really chiral has been further investigated in several papers including [6–13].

where  $F(\rho) = (k_1(u)\rho + k_2(u)) \ln \rho$  with  $k_1(u)$  and  $k_2(u)$  being arbitrary functions of  $u$ .

Indeed this solution has to be compared with the AdS wave solutions in TMG model [14, 15]. Therefore following the observation of [5] one may wonder that the dual theory would be a LCFT. By making use of similarities between NMG and TMG the authors of [27] have computed two point functions of the model and shown that the dual theory is, indeed, a LCFT.

The aim of this paper is to further explore different features of the NMG model. In particular we will study the holographic renormalization of NMG model. More precisely we observe that using the Fefferman-Graham coordinates for the metric for the solution which is asymptotically locally  $AdS_3$ , we do not need any boundary terms to have a well posed variational principle at the critical point. We note, however, that due to the fact that we have a new solution at the critical point one needs to change the asymptotic behavior of the metric to accommodate the new solution. Actually the equations of motion allow us to have a wider class of the boundary condition for the metric as follows

$$g_{ij} = b_{(0)ij} \log(\rho) + g_{(0)ij} + (b_{(2)ij} \log(\rho) + g_{(2)ij}) \rho + \dots . \quad (1.4)$$

Note that when  $b_{(0)ij} \neq 0$  the metric is not asymptotically locally  $AdS_3$ . Nevertheless following [19] for the sufficiently small  $b_{(0)ij}$  one may consider this term perturbatively. Indeed using the AdS/CFT rules,  $b_{(0)ij}$  may be considered as a source for an irrelevant operator in the dual CFT and thus for the small  $b_{(0)ij}$  we could still use the CFT description. Therefore this boundary condition allows two sources for two operators in the boundary CFT. This might be considered as a sign that the dual CFT would be a LCFT.

Using the linearized equations of motion we will find the regularized on-shell action up to quadratic terms. The regularized action can then be used to find the correction functions of the corresponding operators. Our rigorous derivation of the correlation functions based on holographic renormalization is compatible with the results of [27]. Indeed our results confirm that both sectors of the dual theory are indeed two copies of a LCFT with zero central charges.

The paper is organized as follows. In the next section we study the variational principle for the NMG model where we will show that for the solution we are interested in there is no need to add Gibbons-Hawking boundary terms. In section 3 we linearize the equations of motion and solve them perturbatively. In section 4 using the linearized equations we will find the on-shell regularized quadratic action which can be used to read two point functions using the AdS/CFT dictionary. The last section is devoted to the conclusions and discussions. Due to the fact that the computations and the expressions of the equations are very lengthy, the detail of the equations are presented in the several appendices.

## 2 Variational principle and New Massive Gravity

In this section we will study the variational principle for NMG model<sup>3</sup>. To proceed we note that, in general, when we consider the variation of a gravitational action with respect to the

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<sup>3</sup>The variational principle for the NMG model has also been studied in [28].

metric schematically we get the following form for the variation of the action

$$\delta S = \frac{1}{16\pi G} \int_M d^{d+1}x \sqrt{-G} \left[ (\dots) \delta G_{\mu\nu} \right] + \frac{1}{16\pi G} \int_{\partial M} d^d x \sqrt{-\gamma} \left[ (\dots) \delta G_{\mu\nu} + (\dots) \delta G_{\mu\nu, \sigma} \right], \quad (2.1)$$

where  $G_{\mu\nu}$  is the bulk metric while  $\gamma_{\mu\nu}$  is the metric on the boundary  $\partial M$ . As usual setting the first term (the volume term) to zero one finds the equations of motion whereas the boundary terms is set to zero by a proper boundary condition. Indeed the second term in the equation (2.1) is set to zero by imposing Dirichlet boundary condition at the boundary  $\delta G_{\mu\nu}|_{\partial M} = 0$ . On the other hand to have a well-posed variational principle with the Dirichlet boundary condition one has to add a boundary term to the action to remove the last term in (2.1). In general it is difficult to this boundary term for a generic gravitational action, though for the cosmological Einstein-Hilbert action the corresponding term is known; Gibbons-Hawking term. Since in NMG we have higher derivative terms, a priori, it is not clear how to make the variational principle well-posed with the Dirichlet boundary condition. Nevertheless using a specific solution of the equations of motion we will show how to overcome the problem.

Starting with the action of NMG model (1.1) one has

$$\begin{aligned} \delta S = & \frac{1}{16\pi G} \int_R d^3x \sqrt{-G} \left( R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \lambda G_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} \right) \delta G^{\mu\nu} \\ & + \frac{1}{16\pi G} \int_{\partial R} d^2x \sqrt{-\gamma} n_\mu \left\{ G^{\alpha\beta} \delta \Gamma_{\alpha\beta}^\mu - G^{\alpha\mu} \delta \Gamma_{\alpha\beta}^\beta - \frac{1}{m^2} \left[ (2R^{\alpha\beta} - \frac{3}{4} R G^{\alpha\beta}) \delta \Gamma_{\alpha\beta}^\mu \right. \right. \\ & \left. \left. - (2R^{\alpha\mu} - \frac{3}{4} R G^{\alpha\mu}) \delta \Gamma_{\alpha\beta}^\beta - (2\nabla_\beta R^{\alpha\mu} G^{\beta\nu} - \nabla_\beta R^{\alpha\nu} G^{\mu\beta}) \delta G_{\alpha\nu} \right] \right\}, \quad (2.2) \end{aligned}$$

where

$$\begin{aligned} K_{\mu\nu} = & 2\nabla^2 R_{\mu\nu} - \frac{1}{2} (\nabla_\mu \nabla_\nu R + G_{\mu\nu} \nabla^2 R) - 8R_\mu^\sigma R_{\sigma\nu} + \frac{9}{2} R R_{\mu\nu} \\ & + G_{\mu\nu} \left( 3R^{\alpha\beta} R_{\alpha\beta} - \frac{13}{8} R^2 \right) \end{aligned} \quad (2.3)$$

Setting the volume term to zero one finds

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \lambda G_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} = 0, \quad (2.4)$$

which is the equations of motion whose trace is given by

$$R = -\frac{1}{m^2} \left( R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right) + 6\lambda \quad (2.5)$$

It is easy to see that the model admits an AdS vacuum whose radius,  $l$ , is given via the following expression

$$\lambda = -\frac{1}{l^2} \left( 1 + \frac{1}{4l^2 m^2} \right). \quad (2.6)$$

To explore the validity of the variational principle, we will consider a variation of the metric above a vacuum solution which is asymptotically locally  $AdS_3$ . By making use of the

Fefferman-Graham coordinates the metric of an asymptotically locally  $AdS_3$  may be recast to the following form <sup>4</sup>

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij} dx^i dx^j. \quad (2.7)$$

In this notation the boundary terms of the variation of the action (2.2) read

$$\begin{aligned} \delta S_{boundary} = \frac{1}{16\pi G} \int_{\partial R} d^2x \sqrt{-\gamma} \left\{ B^{ij} \delta g_{ij} + \left(2 - \frac{1}{m^2}\right) g^{ij} \delta g'_{ij} - \frac{1}{m^2} \tilde{B}^{ij} g'_{ij} \right. \\ \left. + \frac{1}{m^2} \left[ \rho g^{im} (\nabla^n g'_{nm} - \nabla_m \text{tr}(g^{-1} g')) g^{jk} \right] \delta g_{kj,i} \right\} \end{aligned} \quad (2.8)$$

where

$$\begin{aligned} \tilde{B}^{ij} = & \rho R(g) g^{ij} + 2\rho g^{ij} \text{tr}(g^{-1} g') - 4\rho^2 (g^{-1} g'' g^{-1})^{ij} - 2\rho^2 (g^{-1} g' g^{-1})^{ij} \text{tr}(g^{-1} g') \\ & + 4\rho^2 (g^{-1} [g' g^{-1} g'] g^{-1})^{ij} - 2\rho^2 \text{tr}(g^{-1} g'') g^{ij} + 2\rho^2 g^{ij} \text{tr}(g^{-1} g' g^{-1} g'), \end{aligned} \quad (2.9)$$

and the expression for  $B^{ij}$  is given in the appendix A.

Taking into account that the boundary has no boundary the last term gives a new contribution to the first term ( *i.e.*  $B^{ij}$ ) which altogether do not contribute to the boundary due to the fact that we impose Dirichlet boundary condition at the boundary  $\delta G_{\mu\nu}|_{\partial M} = 0$ . On the other hand using the expression for the asymptotically locally  $AdS_3$  solution,

$$g_{ij} = g_{(0)ij} + (b_{(2)ij} \log(\rho) + g_{(2)ij}) \rho + \dots, \quad (2.10)$$

one observes that  $\tilde{B}^{ij}$  goes to zero as we approach the boundary. In other words this term never reach the boundary. Therefore we are just left with the second term which can be canceled by making use of the standard Gibbons-Hawking term with a proper coefficient. In particular for  $m^2 = 1/2$  this term is zero that means the model does not need boundary term. Thus the variational principle is automatically well-posed for the model.

We note, however, that at the special value of  $m^2 = 1/2$  the model admits a new vacuum solution [26] which is not asymptotically locally  $AdS_3$ . To accommodate this solution one needs to change the asymptotic behavior of the metric as follows

$$g_{ij} = b_{(0)ij} \log(\rho) + g_{(0)ij} + (b_{(2)ij} \log(\rho) + g_{(2)ij}) \rho + \dots. \quad (2.11)$$

It is important to note that with this new term  $\tilde{B}^{ij}$  does not vanish as we approach the boundary. It is worth mentioning that in the context of the AdS/CFT correspondence the new log term corresponds to a perturbation of the dual CFT with an irrelevant operator. Therefore adding this term would destroy the conformal symmetry at UV. Nevertheless following [19] we will assume that  $b_{(0)ij}$  is sufficiently small and this term can be treated perturbatively. Thus even for the case of  $b_{(0)ij} \neq 0$ , we could still work for a solution which is asymptotically locally  $AdS_3$ .

As the conclusion for both  $b_{(0)ij} = 0$  and  $b_{(0)ij} \neq 0$  cases the variational principle is well defined for NMG model without adding any Gibbons-Hawking term for the special value  $m^2 = 1/2$  which is the case we will consider in this paper.

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<sup>4</sup>From now on we set  $l = 1$ .

### 3 Linearized analysis

The main purpose of this article is to compute the correlation functions of the energy momentum tensor of the CFT theory which is dual to the log gravity in NMG model using the AdS/CFT correspondence. To do this one needs to find the on-shell action which can be identified with the generating function for the energy momentum tensor of the dual CFT. We note, however, that it is difficult to solve the equations of motion above an  $AdS$  vacuum in NMG model exactly. Nevertheless since in the present paper we are only interested in two point functions it is enough to solve the equations of motion at the linearized level. Note that in what follows we will also assume that  $b_{(0)ij}$  is sufficiently small such that we neglect higher powers of  $b_{(0)ij}$ . In other words we will linearize the equations around an  $AdS_3$  vacuum whose metric is parametrized by

$$d^2S = \frac{d^2\rho}{4\rho^2} + \frac{1}{\rho}\eta_{ij}dx^i dx^j. \quad (3.1)$$

With this notation we will consider the following perturbation

$$d^2S = \frac{d^2\rho}{4\rho^2} + \frac{1}{\rho}(\eta_{ij} + h_{ij})dx^i dx^j \quad (3.2)$$

#### 3.1 Linearized equations of motion

In this subsection we would like to linearize the equations of motion in the case of  $m^2 = 1/2$  around the  $AdS_3$  vacuum solution where we have  $R = -6$ .<sup>5</sup> In other words we would like to study space times which are asymptotically locally  $AdS_3$ . Taking into account that in this case  $\lambda = -3/2$  the equations of motion can be recast to the following form

$$E_{\mu\nu} \equiv R_{\mu\nu} + \frac{3}{2}G_{\mu\nu} - 2\nabla^2 R_{\mu\nu} + 8R_{\mu}^{\rho}R_{\nu\rho} + 27R_{\mu\nu} - 3G_{\mu\nu}(R^{\rho\sigma}R_{\rho\sigma}) + \frac{117}{2}G_{\mu\nu} = 0. \quad (3.3)$$

It is straightforward, though tedious to plug the metric (3.2) into the above equations of motion and keep only the linearized terms. Doing so one arrives at<sup>6</sup>

$$\begin{aligned} E_{\rho\rho} &= 4\rho^2 tr(h''''') + 16\rho tr(h''') - 12tr(h'') + \rho\partial^i\partial_i tr(h'') + \frac{4}{\rho}tr(h') + 2\partial^i\partial_i tr(h') \\ &\quad + \frac{6}{\rho^2}tr(h) + \frac{2}{\rho}\tilde{R}(h) - 2\partial^i\partial^j h'_{ij} = 0, \\ E_{\rho i} &= -4\rho^2\partial^j h'''_{ji} + 4\rho^2\partial_i tr(h''') - 8\rho\partial^j h''_{ji} + 8\rho\partial_i tr(h'') - \rho\partial^n\partial_n\partial^j h'_{ji} - \partial_i\tilde{R}(h) \\ &\quad + \rho\partial^j\partial_j\partial_i tr(h') - 2\partial_i tr(h') = 0, \\ E_{ij} &= 16\rho^3 h''''_{ij} + 64\rho^2 h'''_{ij} - 8\rho^2\eta_{ij}tr(h''') + 32\rho h''_{ij} - 56\rho\eta_{ij}tr(h'') - 4\rho^2\eta_{ij}\tilde{R}(h'') \\ &\quad + 4\rho^2\partial^m\partial_m h''_{ij} + 24\eta_{ij}tr(h') - 2\rho\eta_{ij}\partial^m\partial_m tr(h') + 4\rho\partial_i\partial^n h'_{nj} + 4\rho\partial_j\partial^n h'_{ni} \\ &\quad - 8\rho\partial_i\partial_j tr(h') - 8\rho\eta_{ij}\tilde{R}(h') + 12\eta_{ij}\tilde{R}(h) - \rho\eta_{ij}\partial^m\partial_m\tilde{R}(h) + \frac{24}{\rho}\eta_{ij}tr(h) = 0. \end{aligned} \quad (3.4)$$

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<sup>5</sup>See appendix C for arbitrary  $m$ .

<sup>6</sup> $\tilde{R}$  denotes the linearized curvature.

On the other hand the linearization of the trace condition,  $R = -6$ , leads to the following equation

$$-4\rho \operatorname{tr}(h'') + \tilde{R}(h) + 2 \operatorname{tr}(h') = 0. \quad (3.5)$$

### 3.2 Near boundary solution of the linearized equations

In this subsection we would like to solve the linearized equations of motion order by order around  $\rho = 0$  (near boundary). To proceed, motivated by the exact solutions of the NMG model we consider the following expansion for the perturbation of the metric

$$h_{ij} = b_{(0)ij} \log(\rho) + g_{(0)ij} + (b_{(2)ij} \log(\rho) + g_{(2)ij})\rho + \cdots. \quad (3.6)$$

As we have already mentioned in what follows we will also assume that  $b_{(0)ij}$  is sufficiently small and therefore the irrelevant deformation can be treated perturbatively.

From the above expansion one has

$$\begin{aligned} h'_{ij} &= b_{(0)ij} \frac{1}{\rho} + b_{(2)ij} \log(\rho) + b_{(2)ij} + g_{(2)ij} + \cdots, \\ h''_{ij} &= -b_{(0)ij} \frac{1}{\rho^2} + b_{(2)ij} \frac{1}{\rho} + \cdots, \\ h'''_{ij} &= b_{(0)ij} \frac{2}{\rho^3} - b_{(2)ij} \frac{1}{\rho^2} + \cdots, \\ h''''_{ij} &= -b_{(0)ij} \frac{6}{\rho^4} + b_{(2)ij} \frac{2}{\rho^3} + \cdots. \end{aligned} \quad (3.7)$$

Plugging these expressions into the linearized equations of motion (3.4) as well as the linearized trace equation (3.5) one can solve the resultant equations order by order leading to algebraic equations for the parameters of the solutions. These equations can be solved for the parameters  $b_{(0)ij}, g_{(0)ij}, b_{(2)ij}$  and  $g_{(2)ij}$ .

To solve the equations we find that at orders  $O(\frac{\log(\rho)}{\rho^2}), O(\frac{1}{\rho^2}), O(\frac{\log(\rho)}{\rho})$  and  $O(\frac{1}{\rho})$  non-trivial relations can be obtained only from the  $E_{\rho\rho}$  equation which are respectively given by

$$\operatorname{tr}(b_{(0)}) = 0, \quad \operatorname{tr}(g_{(0)}) = 0, \quad 5\operatorname{tr}(b_{(2)}) + \tilde{R}[b_{(0)}] = 0, \quad 6\operatorname{tr}(b_{(2)}) - 5\operatorname{tr}(g_{(2)}) - \tilde{R}[g_{(0)}] = 0. \quad (3.8)$$

On the other hand at the order  $O(\log(\rho))$  we get non-trivial relations from trace equation,  $E_{\rho i}$  and  $E_{ij}$  as follows

$$2\operatorname{tr}(b_{(2)}) + \tilde{R}[b_{(0)}] = 0, \quad \partial_i[\operatorname{tr}(b_{(2)}) + \frac{1}{2}\tilde{R}[b_{(0)}]] = 0, \quad 4\operatorname{tr}(b_{(2)}) + \tilde{R}[b_{(0)}] = 0. \quad (3.9)$$

These equations can be solved to find

$$\operatorname{tr}(b_{(0)}) = \operatorname{tr}(g_{(0)}) = \operatorname{tr}(b_{(2)}) = \tilde{R}[b_{(0)}] = 0. \quad (3.10)$$

Using these results, at order  $O(1)$  one finds

$$\text{from } \operatorname{tr} : \operatorname{tr}(g_{(2)}) + \frac{1}{2}\tilde{R}[g_{(0)}] = 0,$$

$$\begin{aligned}
&\text{from } E_{\rho\rho} : \partial^m \partial^n b_{(2)mn} = 0, \\
&\text{from } E_{\rho i} : 4\partial^k b_{(2)ki} + 2\partial_i \text{tr}(g_{(2)}) + \partial^n \partial_n \partial^k b_{(0)ki} - \partial_i \tilde{R}[g_{(0)}] = 0, \\
&\text{from } E_{ij} : -12\eta_{ij} \text{tr}(g_{(2)}) - 3\eta_{ij} \tilde{R}[g_{(0)}] + \partial^m \partial_m b_{(0)ij} + \partial_i \partial^n b_{(0)nj} - \partial_j \partial^n b_{(0)ni} = 0,
\end{aligned} \tag{3.11}$$

while at the order  $O(\rho \log(\rho))$  we get

$$\begin{aligned}
&\text{from } \text{tr} : \tilde{R}[b_{(2)}] = 0, \\
&\text{from } E_{\rho i} : -\partial^n \partial_n \partial^k b_{(2)ki} - \partial_i \tilde{R}[b_{(2)}] = 0, \\
&\text{from } E_{ij} : 4\eta_{ij} \tilde{R}[b_{(2)}] + 4\partial_i \partial^n b_{(2)nj} + 4\partial_j \partial^n b_{(2)ni} = 0.
\end{aligned} \tag{3.12}$$

Finally at the order  $O(\rho)$  one has

$$\begin{aligned}
&\text{from } \text{tr} : \tilde{R}[g_{(2)}] = 0, \\
&\text{from } E_{\rho i} : \partial^n \partial_n [\partial_i \text{tr}(g_{(2)}) - \partial^k b_{(2)ki} - \partial^k g_{(2)ki}] - \partial_i \tilde{R}[g_{(2)}] = 0, \\
&\text{from } E_{ij} : -12\eta_{ij} \tilde{R}[b_{(2)}] + 4\eta_{ij} \tilde{R}[g_{(2)}] - \eta_{ij} \partial^m \partial_m \tilde{R}[g_{(0)}] - 2\eta_{ij} \partial^m \partial_m \text{tr}(g_{(2)}) \\
&\quad + 4\partial^m \partial_m b_{(2)ij} + 4\partial_i \partial^m b_{(2)mj} + 4\partial_j \partial^m g_{(2)mj} + 4\partial_j \partial^m b_{(2)mi} + 4\partial_j \partial^m g_{(2)mi} = 0.
\end{aligned} \tag{3.13}$$

These equations can be solved leading to

$$\text{tr}(g_{(2)}) = 0 = \tilde{R}[g_{(0)}] = \tilde{R}[b_{(2)}] = \tilde{R}[g_{(2)}] = 0, \tag{3.14}$$

and

$$\begin{aligned}
&\partial^m [b_{(2)mi} + \frac{1}{4} \partial^n \partial_n b_{(0)mi}] = 0, \quad \partial^m \partial^n b_{(2)mn} = 0, \quad \partial^m \partial_m \partial^n b_{(2)ni} = 0, \\
&\partial^m \partial_m \partial^n g_{(2)ni} = 0, \quad \partial_i \partial^m b_{(2)mj} + \partial_j \partial^m b_{(2)mi} = 0, \\
&\partial^m \partial_m b_{(0)ij} = \partial_i \partial^n b_{(0)nj} + \partial_j \partial^n b_{(0)ni}, \\
&\partial^m \partial_m b_{(2)ij} = -\partial_i \partial^n b_{(2)nj} - \partial_j \partial^n b_{(2)ni} - \partial_i \partial^n g_{(2)nj} - \partial_j \partial^n g_{(2)ni}.
\end{aligned} \tag{3.15}$$

Note that from the near boundary solution one can not fix the metric completely. Indeed to find a solution of the linearized equations of motion one has to solve them exactly, as we will do in section 4.2.

## 4 Correlation functions

In this section we will compute two point functions of the energy momentum tensor of the CFT which is supposed to be dual to the three dimensional log gravity given by the AdS wave solution of NMG model at the critical value  $m^2 = 1/2$ . To do so, following the AdS/CFT correspondence one needs to identify the bulk on-shell action as the generating function of the CFT. On other hand since the asymptotic behavior of the metric has two divergent terms, one may think of these divergent terms as the sources for the operators in the dual CFT. Since we are only interested in two point functions it is enough to know the on-shell action up to quadratic terms. Then by functional variation with respect to the sources we get the corresponding correlation functions.



## 4.1 On-shell action and counterterms

In this subsection we will evaluate the on-shell action up to quadratic terms. We note, however, that in general the on-shell action is divergent and one needs to regularized it by adding a proper counterterms. Actually taking into account that at the critical value  $m^2 = 1/2$  the Gibbons-Hawking boundary term is not needed, it is enough to linearized the original action (1.1) up to quadratic terms. Indeed it is straightforward to compute the corresponding on-shell action which turns out to be<sup>7</sup>

$$S_{(2)} = \frac{1}{32\pi G} \int_{\partial R} d^2x \left[ -16\rho h''_{ij} h^{ij} + 8\rho^2 h''_{ij} h'^{ij} - 8\rho^2 h'''_{ij} h^{ij} - 4\rho \eta^{im} (\partial^j \partial^n h'_{mn}) h_{ij} \right. \\ \left. + 2\rho \eta^{nl} \eta_{ij} (\partial_n \partial^m h'_{ml}) h^{ij} \right]. \quad (4.1)$$

Note that raising and lowering indices are done with  $g_{(0)ij}$ , and the traces are also taken by  $g_{(0)}$ .

Using the explicit form of the perturbation,  $h_{ij}$ , taken into account that

$$h_{ij} = b_{(0)ij} \log(\rho) + g_{(0)ij} + b_{(2)ij} \rho \log(\rho) + g_{(2)ij} \rho + \dots, \quad (4.2)$$

one finds that the action have a logarithmic divergence as follows

$$S_{(2)} = \frac{1}{32\pi G} \int_{\partial R} d^2x (16b_{(0)}^{ij} b_{(2)ij}) \log(\rho) + \dots. \quad (4.3)$$

Therefore one needs to add a counterterms to cancel this divergence to make the action finite. Note that to maintain the covariance form of the action the counterterm has to be written in terms of  $h_{ij}$  and its derivative. It is easy to see that in our case the corresponding counterterm is given by

$$S_{c.t} = \frac{1}{32\pi G} \int_{\partial R} d^2x (-8\rho h'^{ij} h'_{ij}), \quad (4.4)$$

which in terms of the extrinsic curvature may be recast to the following form

$$S_{c.t} = \frac{1}{32\pi G} \int_{\partial R} d^2x \sqrt{-\gamma} (-8K_{ij}[h] K^{ij}[h]). \quad (4.5)$$

Here  $\gamma_{ij} = \frac{\eta_{ij}}{\rho}$  is the background induced metric. Note that in writing the above expression we have utilized the fact that the the extrinsic curvature is given by  $K_i^j = -\delta_i^j + \rho h_i'^j$ .

As a conclusion the total action up to the quadratic terms is

$$S_{(2),tot} = \frac{1}{32\pi G} \int_{\partial R} d^2x \left[ -16\rho h''_{ij} h^{ij} + 8\rho^2 h''_{ij} h'^{ij} - 8\rho^2 h'''_{ij} h^{ij} - 4\rho \eta^{im} (\partial^j \partial^n h'_{mn}) h_{ij} \right. \\ \left. + 2\rho \eta^{nl} \eta_{ij} (\partial_n \partial^m h'_{ml}) h^{ij} - 8\rho h'_{ij} h'^{ij} \right]. \quad (4.6)$$

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<sup>7</sup>For the detail of the derivation see the appendix D.

It is now straightforward to vary the action with respect to the sources to find the one point functions. Note that since the action contains higher derivative terms, the boundary condition is given by the metric and its first derivative. Therefore we should vary the action with respect to the metric  $h_{ij}$  as well as its derivative  $h'_{ij}$

$$\begin{aligned}\frac{\delta S_{2,tot}}{\delta h^{ij}} &= \frac{1}{16\pi G} \left[ -8\rho h''_{ij} - 4\rho^2 h'''_{ij} - 2\rho\eta_{im}(\partial_j\partial_n h'^{mn}) + \rho\eta^{nl}\eta_{ij}(\partial_n\partial^m h'_{ml}) \right], \\ \frac{\delta S_{2,tot}}{\delta h'^{ij}} &= \frac{1}{16\pi G} \left[ 4\rho^2 h''_{ij} - 4\rho h'_{ij} - 2\rho\eta_{im}\partial_n\partial_j h^{mn} - \rho\eta_{kj}\eta_{mn}\partial^k\partial_i h^{mn} \right].\end{aligned}\quad (4.7)$$

By making use of the explicit form of the perturbation one arrives at

$$\begin{aligned}\frac{\delta S_{2,tot}}{\delta h^{ij}} &= \frac{1}{16\pi G} \left[ -4b_{(2)ij} - 2\eta_{im}\partial_j\partial_n b^{mn}_{(0)} + \eta^{nl}\eta_{ij}(\partial_n\partial^m b_{(0)ml}) \right], \\ \frac{\delta S_{2,tot}}{\delta h'^{ij}} &= \frac{1}{16\pi G} \left[ -4\rho g_{(2)ij} - 4b_{(2)ij}\rho\log(\rho) - 2\eta_{im}(\partial_n\partial_j b^{mn}_{(0)})\rho\log(\rho) \right. \\ &\quad \left. - 2\rho\eta_{im}(\partial_n\partial_j g^{mn}_{(0)}) - \eta_{kj}\eta_{mn}(\partial^k\partial_i g^{mn}_{(0)})\rho - \eta_{kj}\eta_{mn}(\partial^k\partial_i b^{mn}_{(0)})\rho\log(\rho) \right]\end{aligned}\quad (4.8)$$

Therefore we find

$$\begin{aligned}\langle T_{ij} \rangle &= \lim_{\rho \rightarrow 0} \frac{4\pi}{\sqrt{-\eta}} \frac{\delta S_{2,tot}}{\delta h^{ij}} = \frac{1}{4G} \left[ -4b_{(2)ij} - 2\eta_{im}\partial_j\partial_n b^{mn}_{(0)} + \eta^{nl}\eta_{ij}(\partial_n\partial^m b_{(0)ml}) \right], \\ \langle t_{ij} \rangle &= \lim_{\rho \rightarrow 0} \left( \frac{4\pi}{\rho\sqrt{-g}} \frac{\delta I}{\delta h'^{ij}} - \log(\rho) \frac{4\pi}{\sqrt{-\eta}} \frac{\delta I}{\delta h^{ij}} \right) \\ &= \frac{1}{4G} \left[ -4g_{(2)ij} - 2\eta_{im}(\partial_n\partial_j g^{mn}_{(0)}) \right].\end{aligned}\quad (4.9)$$

## 4.2 Holomorphic correlation functions

The goal of this subsection is to compute two point functions of the dual CFT. This can be done by varying the one point functions with respect to the sources. Therefore we need to know the explicit expressions of the one point functions in terms of the sources. The corresponding expressions can be found by solving the linearized equations of motion exactly. To do this it is useful to make the following change of coordinates (see appendix E for definition of our notations)

$$z = x - t \quad \bar{z} = x + t. \quad (4.10)$$

In this notation the linearized equations of motion, (3.4), read

$$\begin{aligned}E_{\rho\rho} &= 2\rho^4 h''''_{z\bar{z}} + 8\rho^3 h'''_{z\bar{z}} - 6\rho^2 h''_{z\bar{z}} + 2\rho^3 \partial\bar{\partial}h''_{z\bar{z}} + 2\rho h'_{z\bar{z}} + 2\rho^2 \partial\bar{\partial}h'_{z\bar{z}} \\ &\quad + 3h_{z\bar{z}} + \rho\partial^2 h_{z\bar{z}} + \rho\bar{\partial}^2 h_{z\bar{z}} - 2\rho\partial\bar{\partial}h_{z\bar{z}} - \rho^2\partial^2 h'_{z\bar{z}} - \rho^2\bar{\partial}^2 h'_{z\bar{z}} = 0, \\ E_{\rho 1} &= -2\rho^2 \partial h'''_{z\bar{z}} - 2\rho^2 \bar{\partial} h'''_{z\bar{z}} + 2\rho^2 \partial h''_{z\bar{z}} + 2\rho^2 \bar{\partial} h''_{z\bar{z}} - 4\rho\partial h''_{z\bar{z}} - 4\rho\bar{\partial} h''_{z\bar{z}} \\ &\quad + 4\rho\partial h'_{z\bar{z}} + 4\rho\bar{\partial} h'_{z\bar{z}} - 2\rho\partial^2 \bar{\partial} h'_{z\bar{z}} - 2\rho\partial\bar{\partial}^2 h'_{z\bar{z}} + 2\rho\partial^2 \bar{\partial} h'_{z\bar{z}} + 2\rho\partial\bar{\partial}^2 h'_{z\bar{z}} \\ &\quad - \partial^3 h_{z\bar{z}} - \partial\bar{\partial}^2 h_{z\bar{z}} + 2\partial^2 \bar{\partial} h_{z\bar{z}} - \partial^2 \bar{\partial} h_{z\bar{z}} - \bar{\partial}^3 h_{z\bar{z}} + 2\partial\bar{\partial}^2 h_{z\bar{z}}\end{aligned}$$

$$-2\partial h'_{z\bar{z}} - 2\bar{\partial} h'_{z\bar{z}} = 0,$$

$$\begin{aligned} E_{\rho 2} = & -2\rho^2 \partial h'''_{z\bar{z}} + 2\rho^2 \bar{\partial} h'''_{z\bar{z}} - 2\rho^2 \partial h'''_{z\bar{z}} + 2\rho^2 \bar{\partial} h'''_{z\bar{z}} - 4\rho \partial h''_{z\bar{z}} + 4\rho \bar{\partial} h''_{z\bar{z}} \\ & - 4\rho \partial h''_{z\bar{z}} + 4\rho \bar{\partial} h''_{z\bar{z}} - 2\rho \partial^2 \bar{\partial} h'_{z\bar{z}} + 2\rho \partial^2 \bar{\partial} h'_{z\bar{z}} - 2\rho \partial^2 \bar{\partial} h'_{z\bar{z}} + 2\rho \partial^2 \bar{\partial} h'_{z\bar{z}} \\ & + \partial^3 h_{z\bar{z}} + \partial \bar{\partial}^2 h_{z\bar{z}} - 2\partial^2 \bar{\partial} h_{z\bar{z}} - \partial^2 \bar{\partial} h_{z\bar{z}} - \bar{\partial}^3 h_{z\bar{z}} + 2\partial \bar{\partial}^2 h_{z\bar{z}} \\ & + 2\partial h'_{z\bar{z}} - 2\bar{\partial} h'_{z\bar{z}} = 0, \end{aligned}$$

$$\begin{aligned} E_{11} = & \rho^3 h''''_{z\bar{z}} + \rho^3 h''''_{z\bar{z}} + 2\rho^3 h''''_{z\bar{z}} + 4\rho^2 h''''_{z\bar{z}} + 4\rho^2 h''''_{z\bar{z}} + 6\rho^2 h^3_{z\bar{z}} \\ & + 2\rho h''_{z\bar{z}} + 2\rho h''_{z\bar{z}} - 10\rho h''_{z\bar{z}} - \rho^2 \partial^2 h''_{z\bar{z}} - \rho^2 \bar{\partial}^2 h''_{z\bar{z}} + 4\rho^2 \partial \bar{\partial} h''_{z\bar{z}} \\ & + \rho^2 \partial \bar{\partial} h''_{z\bar{z}} + \rho^2 \partial \bar{\partial} h''_{z\bar{z}} + 6h'_{z\bar{z}} - \rho \bar{\partial}^2 h'_{z\bar{z}} - \rho^2 \partial^2 h'_{z\bar{z}} - 4\rho \partial \bar{\partial} h'_{z\bar{z}} \\ & - 2\rho \partial^2 h'_{z\bar{z}} - 2\rho \bar{\partial}^2 h'_{z\bar{z}} + 4\rho \partial \bar{\partial} h'_{z\bar{z}} + 3\partial^2 h_{z\bar{z}} + 3\bar{\partial}^2 h_{z\bar{z}} - 6\partial \bar{\partial} h_{z\bar{z}} \\ & - \rho \partial^3 \bar{\partial} h_{z\bar{z}} - \rho \partial \bar{\partial}^3 h_{z\bar{z}} + 2\rho \partial^2 \bar{\partial}^2 h_{z\bar{z}} + \frac{6}{\rho} h_{z\bar{z}} + \rho \partial^2 h'_{z\bar{z}} + \rho \bar{\partial}^2 h'_{z\bar{z}} \\ & + \rho \partial \bar{\partial} h'_{z\bar{z}} + \rho \partial \bar{\partial} h'_{z\bar{z}} = 0, \end{aligned}$$

$$\begin{aligned} E_{22} = & \rho^3 h''''_{z\bar{z}} + \rho^3 h''''_{z\bar{z}} - 2\rho^3 h''''_{z\bar{z}} + 4\rho^2 h''''_{z\bar{z}} + 4\rho^2 h''''_{z\bar{z}} - 6\rho^2 h^3_{z\bar{z}} \\ & + 2\rho h''_{z\bar{z}} + 2\rho h''_{z\bar{z}} + 10\rho h''_{z\bar{z}} + \rho^2 \partial^2 h''_{z\bar{z}} + \rho^2 \bar{\partial}^2 h''_{z\bar{z}} - 4\rho^2 \partial \bar{\partial} h''_{z\bar{z}} \\ & + \rho^2 \partial \bar{\partial} h''_{z\bar{z}} + \rho^2 \partial \bar{\partial} h''_{z\bar{z}} - 6h'_{z\bar{z}} - \rho \bar{\partial}^2 h'_{z\bar{z}} - \rho^2 \partial^2 h'_{z\bar{z}} + 4\rho \partial \bar{\partial} h'_{z\bar{z}} \\ & + 2\rho \partial^2 h'_{z\bar{z}} + 2\rho \bar{\partial}^2 h'_{z\bar{z}} - 4\rho \partial \bar{\partial} h'_{z\bar{z}} - 3\partial^2 h_{z\bar{z}} - 3\bar{\partial}^2 h_{z\bar{z}} + 6\partial \bar{\partial} h_{z\bar{z}} \\ & + \rho \partial^3 \bar{\partial} h_{z\bar{z}} + \rho \partial \bar{\partial}^3 h_{z\bar{z}} - 2\rho \partial^2 \bar{\partial}^2 h_{z\bar{z}} - \frac{6}{\rho} h_{z\bar{z}} - \rho \partial^2 h'_{z\bar{z}} - \rho \bar{\partial}^2 h'_{z\bar{z}} \\ & + \rho \partial \bar{\partial} h'_{z\bar{z}} + \rho \partial \bar{\partial} h'_{z\bar{z}} = 0, \end{aligned}$$

$$\begin{aligned} E_{12} = & -\rho^3 h''''_{z\bar{z}} + \rho^3 h''''_{z\bar{z}} - 4\rho^2 h''''_{z\bar{z}} + 4\rho^2 h''''_{z\bar{z}} - 2\rho h''_{z\bar{z}} + 2\rho h''_{z\bar{z}} - \rho^2 \partial \bar{\partial} h^2_{z\bar{z}} \\ & + \rho^2 \partial \bar{\partial} h''_{z\bar{z}} + 2\rho \partial^2 h'_{z\bar{z}} - 2\rho \bar{\partial}^2 h'_{z\bar{z}} - \rho \partial \bar{\partial} h'_{z\bar{z}} + \rho \partial \bar{\partial} h'_{z\bar{z}} - \rho \partial^2 h'_{z\bar{z}} \\ & + \rho \bar{\partial}^2 h'_{z\bar{z}} = 0. \end{aligned} \tag{4.11}$$

On the other hand from the linearized trace condition one finds

$$\partial^2 h_{z\bar{z}} + \bar{\partial}^2 h_{z\bar{z}} - 2\partial \bar{\partial} h_{z\bar{z}} - 4\rho h''_{z\bar{z}} + 2h'_{z\bar{z}} = 0. \tag{4.12}$$

By making use of the traceless conditions we have found in the section 3, the above equations may be simplified. Indeed the above trace condition reads

$$\partial^2 h_{z\bar{z}} + \bar{\partial}^2 h_{z\bar{z}} = 0. \tag{4.13}$$

Moreover from the equations of motion we get

$$E_{\rho\rho} = \partial^2 h'_{z\bar{z}} + \bar{\partial}^2 h'_{z\bar{z}} = 0,$$

$$\begin{aligned} E_{\rho 1} = & 2\rho^2 (\partial h'''_{z\bar{z}} + \bar{\partial} h'''_{z\bar{z}}) + 4\rho (\partial h''_{z\bar{z}} + \bar{\partial} h''_{z\bar{z}}) + 2\rho (\partial^2 \bar{\partial} h'_{z\bar{z}} + \partial \bar{\partial}^2 h'_{z\bar{z}}) \\ & + (\partial^3 h_{z\bar{z}} + \bar{\partial}^3 h_{z\bar{z}}) + (\partial \bar{\partial}^2 h_{z\bar{z}} + \partial^2 \bar{\partial} h_{z\bar{z}}) = 0, \end{aligned}$$

$$\begin{aligned} E_{\rho 2} = & 2\rho^2 (\partial h'''_{z\bar{z}} - \bar{\partial} h'''_{z\bar{z}}) + 4\rho (\partial h''_{z\bar{z}} - \bar{\partial} h''_{z\bar{z}}) + 2\rho (\partial^2 \bar{\partial} h'_{z\bar{z}} - \partial \bar{\partial}^2 h'_{z\bar{z}}) \\ & - (\partial^3 h_{z\bar{z}} - \bar{\partial}^3 h_{z\bar{z}}) - (\partial \bar{\partial}^2 h_{z\bar{z}} - \partial^2 \bar{\partial} h_{z\bar{z}}) = 0, \end{aligned}$$

$$E_{11} = E_{22} = \rho^2 (h''''_{z\bar{z}} + h''''_{z\bar{z}}) + 4\rho (h''''_{z\bar{z}} + h''''_{z\bar{z}}) + 2(h''_{z\bar{z}} + h''_{z\bar{z}})$$

$$\begin{aligned}
& +\rho(\partial\bar{\partial}h''_{zz} + \partial\bar{\partial}h''_{\bar{z}\bar{z}}) + (\partial\bar{\partial}h'_{zz} + \partial\bar{\partial}h'_{\bar{z}\bar{z}}) = 0, \\
E_{12} &= \rho^2(h''''_{zz} - h''''_{\bar{z}\bar{z}}) + 4\rho(h'''_{zz} - h'''_{\bar{z}\bar{z}}) + 2(h''_{zz} - h''_{\bar{z}\bar{z}}) \\
& +\rho(\partial\bar{\partial}h''_{zz} - \partial\bar{\partial}h''_{\bar{z}\bar{z}}) + (\partial\bar{\partial}h'_{zz} - \partial\bar{\partial}h'_{\bar{z}\bar{z}}) = 0.
\end{aligned} \tag{4.14}$$

Now the aim is to solve the above differential equations to find  $h_{zz}$  and  $h_{\bar{z}\bar{z}}$ . To proceed we will consider the following linear combinations of the above equations by which we get decoupled equations for  $h_{zz}$  and  $h_{\bar{z}\bar{z}}$ .

$$\begin{aligned}
E_{11} + E_{12} &= \rho^2 h''''_{\bar{z}\bar{z}} + 4\rho h^3_{\bar{z}\bar{z}} + (-\alpha^2 \rho + 2)h''_{\bar{z}\bar{z}} + \partial\bar{\partial}h'_{\bar{z}\bar{z}} = 0, \\
E_{11} - E_{12} &= \rho^2 h''''_{zz} + 4\rho h^3_{zz} + (-\alpha^2 \rho + 2)h''_{zz} + \partial\bar{\partial}h'_{zz} = 0
\end{aligned} \tag{4.15}$$

where  $\alpha = \sqrt{-\partial\bar{\partial}}$ . Note that in the momentum space, we have  $\alpha \geq 0$ .

The most general regular solution of the above equations compatible with our boundary conditions are<sup>8</sup>

$$\begin{aligned}
h_{zz} &= c_1(z, \bar{z}) + c_2(z, \bar{z}) \ln(\rho) + c_3(z, \bar{z}) K(0, 2\alpha\sqrt{\rho}), \\
h_{\bar{z}\bar{z}} &= \bar{c}_1(z, \bar{z}) + \bar{c}_2(z, \bar{z}) \ln(\rho) + \bar{c}_3(z, \bar{z}) K(0, 2\alpha\sqrt{\rho}).
\end{aligned} \tag{4.16}$$

Using the asymptotic behavior of the Bessel function  $K(0, 2\alpha\sqrt{\rho})$  at  $\rho \rightarrow 0$ ,

$$K(0, 2\alpha\sqrt{\rho}) = -\ln(\alpha\sqrt{\rho}) - \gamma + [(1 - \gamma)\alpha^2 - \ln(\alpha\sqrt{\rho})\alpha^2]\rho + \dots, \tag{4.17}$$

one finds<sup>9</sup>

$$\begin{aligned}
h_{zz} &= [c_2(z, \bar{z}) - \frac{1}{2}c_3(z, \bar{z})] \ln(\rho) + [c_1(z, \bar{z}) - (\gamma + \ln(\alpha))c_3(z, \bar{z})] \\
& - \frac{1}{2}[\alpha^2 c_3(z, \bar{z})]\rho \ln(\rho) + \left[ [1 - \gamma - \ln(\alpha)]\alpha^2 c_3(z, \bar{z}) \right] \rho + \dots,
\end{aligned} \tag{4.18}$$

where  $\gamma$  is the Euler-Mascheroni constant. This has to be compared with (3.6). Doing so, one arrives at

$$\begin{aligned}
b_{(0)zz} &= c_2(z, \bar{z}) - \frac{1}{2}c_3(z, \bar{z}), \\
g_{(0)zz} &= c_1(z, \bar{z}) - [\gamma + \ln(\alpha)]c_3(z, \bar{z}), \\
b_{(2)zz} &= -\frac{1}{2}[\alpha^2 c_3(z, \bar{z})], \\
g_{(2)zz} &= [1 - \gamma - \ln(\alpha)]\alpha^2 c_3(z, \bar{z}).
\end{aligned} \tag{4.19}$$

It is useful to define  $c_0(z, \bar{z}) = c_2(z, \bar{z}) - \frac{1}{2}c_3(z, \bar{z})$ . In this notation the above equations can be recast to the following form

$$b_{(0)zz} = c_0(z, \bar{z}),$$

<sup>8</sup> The second solution of the equations is  $I(0, 2\alpha\sqrt{\rho})$  which is not regular at  $\rho \rightarrow \infty$ .

<sup>9</sup>Note that this solution is consistent with  $E_{\rho 1}, E_{\rho 2}$  equations if  $\partial\bar{\partial}\bar{c}_2 = 0$  and  $\partial^3\bar{\partial}^2\bar{c}_0 = 0$ .

$$\begin{aligned}
g_{(0)zz} &= c_1(z, \bar{z}) + 2(\gamma + \ln(\alpha))(c_0(z, \bar{z}) - c_2(z, \bar{z})), \\
b_{(2)zz} &= \alpha^2(c_0(z, \bar{z}) - c_2(z, \bar{z})), \\
g_{(2)zz} &= -2[1 - \gamma - \ln(\alpha)]\alpha^2(c_0(z, \bar{z}) - c_2(z, \bar{z})).
\end{aligned} \tag{4.20}$$

Similarly one gets

$$\begin{aligned}
b_{(0)\bar{z}\bar{z}} &= \bar{c}_0(z, \bar{z}), \\
g_{(0)\bar{z}\bar{z}} &= \bar{c}_1(z, \bar{z}) + 2(\gamma + \ln(\alpha))(\bar{c}_0(z, \bar{z}) - \bar{c}_2(z, \bar{z})), \\
b_{(2)\bar{z}\bar{z}} &= \alpha^2(\bar{c}_0(z, \bar{z}) - \bar{c}_2(z, \bar{z})), \\
g_{(2)\bar{z}\bar{z}} &= -2[1 - \gamma - \ln(\alpha)]\alpha^2(\bar{c}_0(z, \bar{z}) - \bar{c}_2(z, \bar{z})).
\end{aligned} \tag{4.21}$$

On the other hand using the equations (4.13), (4.20) and (4.21) we find

$$\begin{aligned}
c_0(z, \bar{z}) &= -\frac{\partial^2}{\partial^2} \bar{c}_0(z, \bar{z}), & c_2(z, \bar{z}) &= -\frac{\partial^3 \bar{\partial}}{\partial \bar{\partial}^3} \bar{c}_2(z, \bar{z}), \\
c_1(z, \bar{z}) &= -\frac{\partial^2}{\partial^2} \bar{c}_1(z, \bar{z}) - 2(\gamma + \ln(\alpha))\frac{\partial^2}{\partial^2}(\bar{c}_0(z, \bar{z}) - \bar{c}_2(z, \bar{z})) \\
&\quad - 2(\gamma + \ln(\alpha))(c_0(z, \bar{z}) - c_2(z, \bar{z})).
\end{aligned} \tag{4.22}$$

Now we have all ingredients to evaluate two point functions. Note that since NMG is a parity preserving model the both sectors of the dual CFT have the same structure. Therefore it is enough to study only one sector of the dual CFT. Therefore in what follows we only consider the *holomorphic* sector of the CFT.

In the  $z, \bar{z}$  notation using the fact that

$$T_{zz} = \frac{1}{4}T_{11} + \frac{1}{4}T_{22} - \frac{1}{4}T_{12} - \frac{1}{4}T_{21}. \tag{4.23}$$

from the equation (4.9) one finds<sup>10</sup>

$$\langle T_{zz} \rangle = \frac{1}{4G}[-4b_{(2)zz} + 4\partial\bar{\partial}b_{(0)zz}], \quad \langle t_{zz} \rangle = \frac{1}{4G}[-4g_{(2)zz} + 4\partial\bar{\partial}g_{(0)zz}]. \tag{4.24}$$

The two point functions can then be obtained from the general roles of AdS/CFT correspondence;

$$\langle T_{ij\dots} \rangle = i\frac{4\pi}{\sqrt{-g_{(0)}}}\frac{\delta}{\delta g_{(0)}^{ij}}\langle \dots \rangle, \quad \langle t_{ij} \rangle = i\frac{4\pi}{\sqrt{-g_{(0)}}}\frac{\delta}{\delta b_{(0)}^{ij}}\langle \dots \rangle. \tag{4.25}$$

On the other hand we note that

$$\frac{\delta}{\delta b_{(0)}^{zz}} = -\frac{1}{4}\frac{\delta}{\delta b_{(0)\bar{z}\bar{z}}} = -\frac{1}{4}\frac{\delta}{\delta \bar{c}_0}. \tag{4.26}$$

Therefore

$$\frac{\delta}{\delta \bar{c}_0}b_{(2)zz} = -\frac{i}{2\pi}\frac{6}{z^4}, \quad \frac{\delta}{\delta \bar{c}_0}g_{(2)zz} = 2[-1 + \gamma + \ln(\alpha)]\frac{\partial^3}{\partial^3}\delta^2(z, \bar{z}),$$

---

<sup>10</sup>See appendix E.

$$\frac{\delta}{\delta \bar{c}_0}[\partial \bar{\partial} b_{(0)zz}] = \frac{i}{2\pi} \frac{6}{z^4}, \quad \frac{\delta}{\delta \bar{c}_0}[\partial \bar{\partial} g_{(0)zz}] = -2(\gamma + \ln(\alpha)) \frac{\partial^3}{\partial} \delta^2(z, \bar{z}) \quad (4.27)$$

Finally we arrive at<sup>11</sup>

$$\begin{aligned} \langle T_{zz} T_{zz} \rangle &= i \frac{4\pi}{\sqrt{-g_{(0)}}} \frac{\delta}{\delta g_{(0)}^{zz}} \langle T_{zz} \rangle = 0, \\ \langle T_{zz} t_{zz} \rangle &= i \frac{4\pi}{\sqrt{-g_{(0)}}} \frac{\delta}{\delta b_{(0)}^{zz}} \langle T_{zz} \rangle = \frac{6/G}{z^4}, \\ \langle t_{zz} t_{zz} \rangle &= i \frac{4\pi}{\sqrt{-g_{(0)}}} \frac{\delta}{\delta b_{(0)}^{zz}} \langle t_{zz} \rangle = \frac{1}{G} \frac{10 + 24\gamma - 12 \ln(m^2 |z|^2)}{z^4}. \end{aligned} \quad (4.28)$$

which is the main goal of the present paper. Note that the non-logarithmic piece in the  $\langle t_{zz} t_{zz} \rangle$  correlation can be removed by a shift in  $t$  given by  $t_{zz} \rightarrow t_{zz} - \frac{10+24\gamma}{12} T_{zz}$ .

Similarly one can show

$$\langle T_{\bar{z}\bar{z}} T_{\bar{z}\bar{z}} \rangle = 0, \quad \langle T_{\bar{z}\bar{z}} t_{\bar{z}\bar{z}} \rangle = \frac{6/G}{z^4}, \quad \langle t_{\bar{z}\bar{z}} t_{\bar{z}\bar{z}} \rangle = -\frac{12 \ln(m^2 |z|^2)}{G z^4}. \quad (4.29)$$

The other correlations are zero up to a contact term.

## 5 Conclusions

In this paper we have explored the holographic renormalization for NMG model which is a three dimensional parity preserving massive gravity. We have shown that at the critical value  $l^2 m^2 = 1/2$  we do not need any boundary terms to make the variational principle of NMG model well-posed with Dirichlet boundary condition.

Using a proper fall off for the metric as a boundary condition we have computed the regularized on shell action by adding a suitable counterterm. This in turn can be used to find correlation functions of the dual field theory.

Indeed the main goal of this paper was to compute the two point function of the energy momentum tensor of the dual CFT of NMG model at the critical point. Following the previous studies we would have expected that the corresponding correlation function to have a LCFT-like behavior. Actually we have found that the correlation functions, indeed, satisfy the expected expression for a LCFT.

Comparing our results (4.28) and (4.29) with those in a LCFT given by [29, 30]

$$\begin{aligned} \langle T_{zz} T_{zz} \rangle &= \frac{c}{2z^4}, \\ \langle T_{zz} t_{zz} \rangle &= \frac{b}{2z^4}, \\ \langle t_{zz} t_{zz} \rangle &= -\frac{\log(z)[c \log(z) + 2b]}{2z^4}, \end{aligned} \quad (5.1)$$

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<sup>11</sup>We use this relations:  $\frac{1}{\partial \bar{\partial}} \delta^2(z, \bar{z}) = \frac{i}{2\pi} \ln(m^2 |z|^2)$   $\ln(\alpha) \frac{1}{\partial \bar{\partial}} \delta^2(z, \bar{z}) = -\frac{i}{8\pi} \ln^2(m^2 |z|^2)$ .

one observes that the holographic two point functions have expected from with central charge  $c = \bar{c} = 0$  and new anomaly  $b = \bar{b} = \frac{12}{G}$ . Having the same structure for left and right handed sectors with the equal central charges and anomaly reflects the fact that the theory is parity preserving.

## Acknowledgments

We would like to thank H. Afshar, R. Fareghbal, D. Grumiller, A. Mosaffa and S. Rouhani for useful discussions. Special thank to M. Taylor for carefully reading the paper and pointing out a mistake in the early version of the manuscript.

## A Boundary term in the Fefferman-Graham coordinates

$$\begin{aligned}
B^{ij} = & \left( -\frac{3}{\rho} g^{ij} + \rho g'^{ij} \right) - \frac{1}{m^2} \left[ -\frac{1}{2} R(g) g^{ij} + 4(g^{-1} g'' g^{-1})^{ij} + 3\rho (g^{-1} g' g^{-1})^{ij} \text{tr}(g^{-1} g') \right. \\
& - \frac{3}{2\rho} g^{ij} - 6\rho (g^{-1} [g' g^{-1} g'] g^{-1})^{ij} + 6\rho \text{tr}(g^{-1} g'')^{ij} - \frac{5}{2} \rho \text{tr}(g^{-1} g' g^{-1} g') g^{ij} + \frac{17}{4} g'^{ij} \\
& - 2\rho^2 ([g^{-1} g'] g^{-1})^{ij} \text{tr}(g^{-1} g'') + \rho^2 ([g^{-1} g'] g^{-1})^{ij} \text{tr}(g^{-1} g' g^{-1} g') + 4(g^{-1} g' g^{-1})^{ij} \\
& - \rho (g^{-1} g')^{ij} \text{tr}(g^{-1} g') + 2\rho (g^{-1} [g' g^{-1} g'])^{ij} - \rho (g^{-1} g')^{ij} R(g) - \rho (g^{-1} g')^{ij} \text{tr}(g^{-1} g') \\
& + 2\rho^2 (g^{-1} g' g''^{-1})^{ij} - 4\rho^2 (g^{-1} g' [g' g^{-1} g'])^{ij} - 2\rho g'^{ij} \text{tr}(g^{-1} g') - \frac{1}{2} \rho R'(g) g^{ij} - \frac{1}{2} \rho R(g) (g^{-1} g' g^{-1})^{ij} \\
& - \rho (g^{-1} g' g^{-1})^{ij} \text{tr}(g^{-1} g') + 2\rho^2 (g^{-1} g^{(3)} g^{-1})^{ij} + \rho^2 (g^{-1} g'' g^{-1})^{ij} \text{tr}(g^{-1} g') - 2\rho (g^{-1} [g' g^{-1} g'])^{ij} \\
& - 2\rho^2 (g^{-1} [g' g^{-1} g'] g^{-1})^{ij} + \rho^2 (g^{-1} g' g^{-1})^{ij} \partial_\rho \text{tr}(g^{-1} g') - \rho g'^{ij} \text{tr}(g^{-1} g') + 2\rho (g' g^{-1} g')^{ij} \\
& + 2\rho^2 (g^{-1} g' g'^{-1})^{ij} \text{tr}(g^{-1} g') - \rho R(g) g'^{ij} \\
& + 2\rho^2 ([g^{-1} g'] g'')^{ij} + 4\rho^2 (g' g'')^{ij} - \frac{1}{2} \rho R(g) ([g^{-1} g'] g^{-1})^{ij} - 4\rho^2 (g' [g^{-1} g' g'])^{ij} \\
& + 2\rho^2 (g' g')^{ij} \text{tr}(g^{-1} g') + \rho^2 ([g^{-1} g' g'^{-1}])^{ij} \text{tr}(g^{-1} g') + 2\rho (\partial^j g^{in}) (\nabla^k g'_{kn} - \nabla_n \text{tr}(g^{-1} g^{-1} g')) \\
& - \rho ([g^{-1} g'] g^{-1})^{ij} \text{tr}(g^{-1} g') + \rho g^{mn} (\nabla^k g'_{kn} - \nabla_n \text{tr}(g^{-1} g')) g'^{ij}_{,m} \\
& \left. + \rho g^{mn} g^{kj} \Gamma^i_{mk} (\nabla^l g'_{ln} - \nabla_n \text{tr}(g^{-1} g')) + i \leftrightarrow j \right] \tag{A.1}
\end{aligned}$$

## B Equations of motion in the Fefferman-Graham coordinates

In this appendix we will work out how to write the equations of motion in terms of the the Fefferman-Graham coordinates. To do this we need to express different component of the equations of motion including  $R$ ,  $R_{\mu\nu}$ ,  $R^{\mu\nu} R_{\mu\nu}$ ,  $\nabla^2 R_{\mu\nu}$  in terms of the Fefferman-Graham coordinates. It is straightforward, though tedious to show that

$$\begin{aligned}
R &= 4\rho^2 R_{\rho\rho} + \rho g^{ij} R_{ij} \\
&= -6 - 4\rho^2 \text{tr}(g^{-1} g'') + \rho^2 \text{tr}(g^{-1} g' g^{-1} g') + 2\rho \text{tr}(g^{-1} g') - \rho^2 \text{tr}^2(g^{-1} g') \\
&+ 2\rho^2 \text{tr}[g^{-1} (g' g^{-1} g')] + \rho R(g) \tag{B.1}
\end{aligned}$$

$$\begin{aligned}
R_{\rho\rho} &= -\frac{1}{2}\text{tr}(g^{-1}g'') + \frac{1}{4}\text{tr}(g^{-1}g'g^{-1}g') - \frac{1}{2\rho^2} \\
R_{i\rho} &= \frac{1}{2}[\nabla^m g'_{mi} - \nabla_i \text{tr}(g^{-1}g')] \\
R_{ij} &= \frac{1}{2}R(g)g_{ij} + g_{ij}\text{tr}(g^{-1}g') - \rho[2g''_{ij} + g'_{ij}\text{tr}(g^{-1}g') - 2(g'g^{-1}g')_{ij}] - \frac{2}{\rho}g_{ij} \quad (\text{B.2})
\end{aligned}$$

$$\begin{aligned}
R_\rho^\mu R_{\mu\rho} &= \rho^2[\text{tr}(g^{-1}g'')]^2 - \rho^2\text{tr}(g^{-1}g'')\text{tr}(g^{-1}g'g^{-1}g') + 2\text{tr}(g^{-1}g'') - \text{tr}(g^{-1}g'g^{-1}g') + \frac{1}{\rho^2} \\
&+ \frac{1}{4}\rho^2[\text{tr}(g^{-1}g'g^{-1}g')]^2 + \frac{1}{4}\rho g^{ij}[\nabla^m g'_{mi} - \nabla_i \text{tr}(g^{-1}g')][\nabla^n g'_{nj} - \nabla_j \text{tr}(g^{-1}g')] \quad (\text{B.3})
\end{aligned}$$

$$\begin{aligned}
R_\rho^\mu R_{\mu i} &= \left[ -\rho^2\text{tr}(g^{-1}g'') + \frac{1}{2}\rho^2\text{tr}(g^{-1}g'g^{-1}g') - 2 + \frac{1}{4}\rho R(g) + \frac{1}{2}\rho\text{tr}(g^{-1}g') \right] \\
&\times [\nabla^k g'_{ki} - \nabla_i \text{tr}(g^{-1}g')] \\
&+ \left[ -\rho^2(g^{-1}g'') - \frac{1}{2}\rho^2(g^{-1}g')\text{tr}(g^{-1}g') + \rho^2[g^{-1}(g'g^{-1}g')] \right]_i^k \\
&[\nabla^n g'_{nk} - \nabla_k \text{tr}(g^{-1}g')] \quad (\text{B.4})
\end{aligned}$$

$$\begin{aligned}
R_i^\mu R_{\mu j} &= \frac{1}{4}\rho R^2(g)g_{ij} + \rho R(g)g_{ij}\text{tr}(g^{-1}g') - 2\rho^2 R(g)g''_{ij} - \rho^2 R(g)g'_{ij}\text{tr}(g^{-1}g') + \frac{4}{\rho}g_{ij} \\
&+ 2\rho^2 R(g)(g'g^{-1}g')_{ij} - 2R(g)g_{ij} + \rho g_{ij}[\text{tr}(g^{-1}g')]^2 - 4\rho^2 g''_{ij}\text{tr}(g^{-1}g') \\
&- 2\rho^2 g'_{ij}[\text{tr}(g^{-1}g')]^2 + 4\rho^2(g'g^{-1}g')_{ij}\text{tr}(g^{-1}g') - 4g_{ij}\text{tr}(g^{-1}g') + 4\rho^3[g^{-1}g'']_i^k g''_{kj} \\
&+ 4\rho^3[g^{-1}g'']_i^k g'_{kj}\text{tr}(g^{-1}g') - 8\rho^3[g^{-1}g'']_i^k(g'g^{-1}g')_{kj} + 4\rho g''_{ij} + 4\rho[g^{-1}g'']_i^k g_{kj} \\
&+ \rho^3[g^{-1}g']_i^k g'_{kj}[\text{tr}(g^{-1}g')]^2 - 4\rho^3[g^{-1}g']_i^k(g'g^{-1}g')_{kj} + 4\rho g'_{ij}\text{tr}(g^{-1}g') \\
&+ 4\rho^3[g^{-1}(g'g^{-1}g')]_i^k(g'g^{-1}g')_{kj} - 4\rho[g^{-1}(g'g^{-1}g')]_i^k g_{kj} - 4\rho(g'g^{-1}g')_{ij} \\
&+ \rho^2[\nabla^m g'_{mi} - \nabla_i \text{tr}(g^{-1}g')][\nabla^n g'_{nj} - \nabla_j \text{tr}(g^{-1}g')] \quad (\text{B.5})
\end{aligned}$$

$$\begin{aligned}
R^{\mu\nu} R_{\mu\nu} &= 4\rho^4[\text{tr}(g^{-1}g'')]^2 + \rho^4[\text{tr}(g^{-1}g'g^{-1}g')]^2 - 4\rho^4\text{tr}(g^{-1}g'')\text{tr}(g^{-1}g'g^{-1}g') \\
&- 2\rho^3 R(g)\text{tr}(g^{-1}g'') - \rho^3 R(g)[\text{tr}(g^{-1}g')]^2 + 2\rho^3 R(g)\text{tr}[g^{-1}(g'g^{-1}g')] \\
&- 4\rho R(g) + 6\rho^2[\text{tr}(g^{-1}g')]^2 - 4\rho^3\text{tr}(g^{-1}g'')\text{tr}(g^{-1}g') - 2\rho^3[\text{tr}(g^{-1}g')]^3 \\
&+ 4\rho^3\text{tr}(g^{-1}g')\text{tr}[g^{-1}(g'g^{-1}g')] - 8\rho\text{tr}(g^{-1}g') + 4\rho^4[g^{ij}g^{km}g''_{im}g''_{kj}] \\
&+ 4\rho^4 g^{ij}g^{km}g''_{im}g'_{kj}\text{tr}(g^{-1}g') - 8\rho^4 g^{ij}g^{km}g''_{im}(g'g^{-1}g')_{kj} + \frac{1}{2}\rho^2 R^2(g) \\
&+ \rho^4 g^{ij}g^{km}g'_{im}g'_{kj}[\text{tr}(g^{-1}g')]^2 - 4\rho^4 g^{ij}g^{km}g'_{im}(g'g^{-1}g')_{jk}\text{tr}(g^{-1}g') \\
&+ 4\rho^4 g^{ij}g^{km}(g'g^{-1}g')_{im}(g'g^{-1}g')_{jk} - 12\rho^2\text{tr}(g^{-1}g'g^{-1}g') + 16\rho^2\text{tr}(g^{-1}g'') \\
&+ 12 + 2\rho^2 R(g)\text{tr}(g^{-1}g') + 2\rho^3 g^{ij}[\nabla^k g'_{ki} - \nabla_i \text{tr}(g^{-1}g')][\nabla^m g'_{mj} - \nabla_j \text{tr}(g^{-1}g')] \quad (\text{B.6})
\end{aligned}$$

$$\begin{aligned}
\nabla^2 R_{\rho\rho} &= -2\rho^2\partial_\rho\partial_\rho\text{tr}(g^{-1}g'') + \rho^2\partial_\rho\partial_\rho\text{tr}(g^{-1}g'g^{-1}g') - 10\rho\partial_\rho\text{tr}(g^{-1}g'') \\
&+ 4\rho\partial_\rho\text{tr}(g^{-1}g'g^{-1}g') + 2\text{tr}(g^{-1}g'g^{-1}g') - 3\text{tr}(g^{-1}g'') - (g^{-1}g')^{ik}(g^{-1}g')_{ik}
\end{aligned}$$



$$\begin{aligned}
& -\frac{1}{2}\rho g^{ij}\partial_i\partial_j\text{tr}(g^{-1}g'') + \frac{1}{4}\rho g^{ij}\partial_i\partial_j\text{tr}(g^{-1}g'g^{-1}g') + \partial^i[\nabla^m g'_{mi} - \nabla_i\text{tr}(g^{-1}g')] \\
& -2\rho\partial_\rho\text{tr}(g^{-1}g'') - \rho^2\text{tr}(g^{-1}g')\partial_\rho\text{tr}(g^{-1}g'') + \frac{1}{2}\rho^2\text{tr}(g^{-1}g')\partial_\rho\text{tr}(g^{-1}g'g^{-1}g') \\
& -4\rho\text{tr}(g^{-1}g')\text{tr}(g^{-1}g'') + 2\rho\text{tr}(g^{-1}g')\text{tr}(g^{-1}g'g^{-1}g') + \frac{1}{2}\rho g^{ij}\Gamma_{ij}^k\partial_k\text{tr}(g^{-1}g'') \\
& -\frac{1}{4}\rho g^{ij}\Gamma_{ij}^k\partial_k\text{tr}(g^{-1}g'g^{-1}g') - g^{ij}\Gamma_{ij}^m[\nabla^n g'_{nm} - \nabla_m\text{tr}(g^{-1}g')] + \frac{1}{\rho}\text{tr}(g^{-1}g') \\
& +\frac{1}{2}\rho g^{ij}\Gamma_{ij}^k(g^{-1}g')_k^m[\nabla^n g'_{nm} - \nabla_m\text{tr}(g^{-1}g')] + \rho(g^{-1}g')^{ij}g'_{ij}\text{tr}(g^{-1}g') \\
& +\rho^2[(g^{-1}g')^{ik}g'_{ki}\text{tr}(g^{-1}g'')] - \frac{1}{2}\rho^2[(g^{-1}g')^{ik}g'_{ki}\text{tr}(g^{-1}g'g^{-1}g')] + (g^{-1}g')^{ik}g'_{ki} \\
& +\frac{1}{2\rho}R(g) - \frac{1}{2}R(g)\text{tr}(g^{-1}g') - \frac{3}{2}[\text{tr}(g^{-1}g')]^2 + 2\rho(g^{-1}g')^{ij}g''_{ij} \\
& -2\rho(g^{-1}g')_i^j(g'g^{-1}g')_j^i + \frac{1}{4}\rho R(g)(g^{-1}g')_i^j(g^{-1}g')_j^i + \frac{1}{2}\rho(g^{-1}g')_i^j(g^{-1}g')_j^i\text{tr}(g^{-1}g') \\
& -\rho^2(g^{-1}g')^{ik}(g^{-1}g')_i^m g''_{km} - \frac{1}{2}\rho(g^{-1}g')^{ik}\partial_i[\nabla^m g'_{mk} - \nabla_k\text{tr}(g^{-1}g')] \\
& +\rho^2(g^{-1}g')^{ik}(g^{-1}g')_i^m(g'g^{-1}g')_{km} - \frac{1}{2}\rho^2(g^{-1}g')^{ik}(g^{-1}g')_i^m g'_{km}\text{tr}(g^{-1}g') \\
& -\frac{1}{2}\rho g^{ij}\partial_j[(g^{-1}g')_i^k(\nabla^m g'_{mk} - \nabla_k\text{tr}(g^{-1}g'))] + \frac{1}{2}\rho(g^{-1}g')^{ik}\Gamma_{ik}^m[\nabla^n g'_{nm} - \nabla_m\text{tr}(g^{-1}g')] \\
\end{aligned} \tag{B.7}$$

$$\begin{aligned}
\nabla^2 R_{i\rho} = & \left[ 2\rho^2\partial_\rho\partial_\rho + \rho[g^{-1}(g^{-1}g')]^{kj}(g_{jk} - \rho g'_{jk}) + \frac{3}{2}\rho\text{tr}(g^{-1}g') + \rho^2\text{tr}(g^{-1}g')\partial_\rho \right. \\
& + 5\rho\partial_\rho + \left. \frac{1}{2}\rho g^{jm}(\partial_m\partial_j) \right] \left[ \nabla^k g'_{ki} - \nabla_i\text{tr}(g^{-1}g') \right] \\
& + \left[ \rho\delta_i^j\partial_\rho - \frac{1}{2}\rho^2(g^{-1}g')_i^m(g^{-1}g')_m^j - \frac{1}{2}\rho g^{mk}(\partial_m\Gamma_{ik}^j) - \frac{1}{2}\rho g^{mk}\Gamma_{ik}^j\partial_m \right. \\
& - \frac{1}{2}\rho\text{tr}(g^{-1}g')\delta_i^j - \frac{1}{2}(g^{-1}g')_i^j + \frac{1}{2}\rho(g^{-1}[g^{-1}g'])^{mj}(g_{mi} - \rho g'_{mi}) \\
& + \left. \frac{1}{2}\rho^2\text{tr}(g^{-1}g')(g^{-1}g')_i^j - \delta_i^j - \rho^2\partial_\rho(g^{-1}g')_i^j \right] \left[ \nabla^k g'_{kj} - \nabla_j\text{tr}(g^{-1}g') \right] \\
& + \left[ \rho g^{jk}\partial_j(g_{ik} + \rho g'_{ik}) + \rho g^{jk}(g_{ik} - \rho g'_{ik})\partial_j + \rho\partial_i - \rho^2(g^{-1}g')_i^k\partial_k \right] \\
& \times \left[ \text{tr}(g^{-1}g'') - \frac{1}{2}\text{tr}(g^{-1}g'g^{-1}g') + \frac{1}{\rho^2} \right] \\
& + \left[ \frac{1}{2}g^{nk}\partial_k - \frac{1}{2}\rho g^{jk}\partial_j \left[ -\frac{1}{\rho}\delta_k^n + (g^{-1}g')_k^n \right] - \frac{1}{2}\rho g^{jk} \left[ -\frac{1}{\rho}\delta_k^n + (g^{-1}g')_k^n \right]\partial_j \right. \\
& - \frac{1}{2}g^{jk}\Gamma_{jk}^n + \frac{1}{2}g^{nk}\partial_k - \frac{1}{2}\rho[g^{-1}(g^{-1}g')]^{kn}\partial_k \left. \right] \left[ \frac{1}{2}R(g)g_{in} + g_{in}\text{tr}(g^{-1}g') \right. \\
& - 2\rho g''_{in} - \rho g'_{in}\text{tr}(g^{-1}g') + 2\rho(g'g^{-1}g')_{in} - \left. \frac{2}{\rho}g_{in} \right] \\
& - \frac{1}{2}g^{jk}\Gamma_{ik}^n \left[ \frac{1}{2}R(g)g_{jn} + g_{jn}\text{tr}(g^{-1}g') - 2\rho g''_{jn} - \rho g'_{jn}\text{tr}(g^{-1}g') + 2\rho(g'g^{-1}g')_{jn} \right. \\
& - \left. \frac{2}{\rho}g_{jn} \right] - \rho g^{jk}\Gamma_{jk}^m\nabla_m R_{i\rho} - \rho g^{jk}\Gamma_{ij}^m\nabla_k R_{m\rho} + \frac{1}{2}\rho[g^{-1}(g^{-1}g')]_k^j(\Gamma_{ik}^n R_{jm} + \Gamma_{jk}^n R_{in})
\end{aligned}$$

(B.8)

$$\begin{aligned}
\nabla^2 R_{ij} = & \left[ 2 + 4\rho^2 \partial_\rho \partial_\rho + 4\rho \partial_\rho + \rho g^{km} \partial_m \partial_k + (-4\rho + 2\rho^2 \text{tr}(g^{-1}g')) \partial_\rho - \rho g^{km} \Gamma_{km}^n \nabla_n \right] \\
& \left[ \frac{1}{2} R(g) g_{ij} + g_{ij} \text{tr}(g^{-1}g') - 2\rho g_{ij}'' - \rho g_{ij}' \text{tr}(g^{-1}g') + 2\rho (g' g^{-1} g')_{ij} - \frac{2}{\rho} g_{ij} \right] \\
& + \left[ 4\rho \delta_i^n \partial_\rho - 4\rho^2 (g^{-1}g')_i^n \partial_\rho - \delta_i^n - 2\rho (g^{-1}g')_i^n + \rho^2 (g^{-1}g')_i^k (g^{-1}g')_k^n \right. \\
& - \rho g^{mk} \Gamma_{im}^k \nabla_k + \rho \text{tr}(g^{-1}g') \delta_i^n - \rho^2 \text{tr}(g^{-1}g') (g^{-1}g')_i^n - \rho g^{mk} \Gamma_{ik}^n \partial_m \\
& \left. - \rho g^{km} (\partial_m \Gamma_{ik}^n) - 2\rho^2 \partial_\rho (g^{-1}g')_i^n \right] R_{nj} + i \leftrightarrow j \\
& - \left[ \rho g^{km} (g_{im} - \rho g_{im}') \nabla_k + \rho g^{km} \partial_m (g_{ik} - \rho g_{ik}') + \rho g^{km} (g_{ik} - \rho g_{ik}') \partial_m \right] \\
& \times \left[ \nabla^n g_{nj}' \nabla_j \text{tr}(g^{-1}g') \right] + i \leftrightarrow j \\
& - \left[ \rho (g^{-1}g')_j^m R_{im} + \rho (g^{-1}g')_i^n R_{nj} - \rho^2 (g^{-1}g')_i^n (g^{-1}g')_j^m R_{nm} + i \leftrightarrow j \right]. \quad (\text{B.9})
\end{aligned}$$

Having these expressions and plugging them into the equations of motion one can write the equation we were looking for.

## C Linearized equations of motion for arbitrary $m$

In this appendix we find the linearized equations of motion for arbitrary  $m$ . To do this we need to linearized different terms in the equations of motion as follows

$$\begin{aligned}
\nabla^2 R_{\rho\rho} &= -2\rho^2 \text{tr}(h^4) - 8\rho \text{tr}(h^3) + \text{tr}(h'') + \frac{1}{\rho} \text{tr}(h') - \frac{1}{2} \rho \partial^n \partial_n \text{tr}(h'') \\
&\quad + \frac{1}{2\rho} R(h) + \partial^i \partial^j \text{tr}(h'), \\
\nabla^2 R_{\rho i} &= 2\rho^2 \partial^m h_{mi}^{(3)} - 2\rho^2 \partial_i \text{tr}(h^{(3)}) + \frac{\rho}{2} \partial^j \partial_j [\partial^m h'_{mi} - \partial_i \text{tr}(h')] \\
&\quad + \frac{1}{2} \partial_i \tilde{R}(h) + 4\rho \partial^m h''_{mi} - 4\rho \partial_i \text{tr}(h'') + 2\partial_i \text{tr}(h') - \partial^m h'_{mi}, \\
\nabla^2 R_{ij} &= -8\rho^3 h_{ij}^{(4)} - 32\rho^2 h_{ij}^{(3)} + 4\rho^{(2)} \eta_{ij} \text{tr}(h^{(3)}) + 2\rho^2 \eta_{ij} \tilde{R}(h'') \\
&\quad - 12\rho h''_{ij} - 2\eta_{ij} \text{tr}(h') + 4\rho \eta_{ij} \tilde{R}(h') + 4\rho \eta_{ij} \text{tr}(h'') \\
&\quad + \frac{1}{2} \rho \eta_{ij} \partial^m \partial_m \tilde{R}(h) + \rho \eta_{ij} \partial^m \partial_m \text{tr}(h') - 2\rho^2 \partial^m \partial_m h''_{ij} \\
&\quad - \eta_{ij} \tilde{R}(h) - 2\rho \partial_i \partial^m h'_{mj} - 2\rho \partial_j \partial^m h'_{mi} + 4\rho \partial_i \partial_j \text{tr}(h'), \quad (\text{C.1})
\end{aligned}$$

$$(R^{\mu\nu} R_{\mu\nu})^{(1)} = 16\rho^2 \text{tr}(h'') - 8\rho \text{tr}(h') - 8 \text{tr}(h) - 4\rho \tilde{R},$$

$$\begin{aligned}
(R_\rho^\sigma R_{\sigma\rho})^{(1)} &= 2 \operatorname{tr}(h''), \\
(R_i^\sigma R_{\sigma\rho})^{(1)} &= -2\partial^j h'_{ji} + 2\partial_i \operatorname{tr}(h'), \\
(R_i^\sigma R_{\sigma j})^{(1)} &= 8\rho h''_{ij} + 4\rho\eta_{ij} \operatorname{tr}(h') - 2\eta_{ij}\tilde{R}(h) + \frac{4}{\rho}h_{ij},
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
E_{\rho\rho} &= -\frac{1}{2} \operatorname{tr}(h'') + \frac{1}{2m^2}[4\rho^2 \operatorname{tr}(h^{(4)}) + 16\rho \operatorname{tr}(h^{(3)}) - \frac{23}{2} \operatorname{tr}(h'') + \rho\partial^i\partial_i \operatorname{tr}(h'') \\
&\quad + \frac{4}{\rho} \operatorname{tr}(h') + 2\partial^i\partial_i \operatorname{tr}(h') + \frac{6}{\rho^2} \operatorname{tr}(h) + \frac{2}{\rho}R(h) - 2\partial^i\partial^j h'_{ij}] = 0, \\
E_{\rho i} &= \frac{1}{2}(\partial^m h'_{mi} - \partial_i \operatorname{tr}(h')) + \frac{1}{2m^2}[-4\rho^2\partial^m h'_{mi} + 4\rho^2\partial_i \operatorname{tr}(h^3) - \frac{1}{2}\partial^m h'_{mi} - 8\rho\partial^m h''_{mi} \\
&\quad - \frac{3}{2}\partial_i \operatorname{tr}(h')\rho\partial^m\partial_m\partial^n h'_{ni} + \rho\partial^m\partial_m\partial_i \operatorname{tr}(h') - \partial_i\tilde{R}(h) - 8\rho\partial_i \operatorname{tr}(h'')] = 0, \\
E_{ij} &= \frac{1}{2}\tilde{R}(h)\eta_{ij} + \eta_{ij} \operatorname{tr}(h') - 2\rho h''_{ij} - \frac{1}{2\rho}h_{ij} + \frac{1}{2m^2}[34\rho h''_{ij} + 23\eta_{ij} \operatorname{tr}(h') + 16\rho^3 h^{(4)}_{ij} \\
&\quad + 64\rho^2 h^{(3)}_{ij} - 8\rho^2\eta_{ij} \operatorname{tr}(h^{(3)}) - 4\rho^{(2)}\eta_{ij}\tilde{R}(h'') + \frac{23}{2}\eta_{ij}\tilde{R}(h) - 56\rho\eta_{ij} \operatorname{tr}(h'') \\
&\quad - \rho\eta_{ij}\partial^m\partial_m\tilde{R}(h) - 2\rho\eta_{ij}\partial^m\partial_m \operatorname{tr}(h') + 4\rho^2\partial^m\partial_m h''_{ij} + 4\rho\partial_i\partial^m h'_{mj} + 4\rho\partial_j\partial^m h'_{mi} \\
&\quad + \frac{1}{2\rho}h_{ij} - 8\rho\partial_i\partial_j \operatorname{tr}(h') - 8\rho\partial_i\partial_j \operatorname{tr}(h') - 8\rho\eta_{ij}\tilde{R}(h') + \frac{24}{\rho}\eta_{ij} \operatorname{tr}(h)] = 0 \tag{C.3}
\end{aligned}$$

Putting these expressions together one can find the linearized equations of motion.

## D Linearized action up to quadratic terms

To find the on-shell action we need to plug the classical solution of the equations of motion to the complete action. Of course since in our case where  $m^2 = 1/2$  we do not need the Gibbons-Hawking boundary term, the whole contributions to the on-shell action come from the boundary terms given in (2.2). Since we are only interested in the two point functions we will need to know the action up to the quadratic terms. In what follows we will write the contribution of each term to the on-shell action.

$$\int_{\partial R} d^2x \sqrt{-\gamma} n_\mu \left[ G^{\alpha\beta} \delta\Gamma_{\alpha\beta}^\mu - G^{\alpha\mu} \delta\Gamma_{\alpha\beta}^\beta \right] = \frac{1}{2} \int_{\partial R} d^2x \left[ \frac{3}{\rho} h^{ij} h_{ij} + [-3h^{ij}] h'_{ij} \right] \tag{D.1}$$

$$\begin{aligned}
\int_{\partial R} d^2x \sqrt{-\gamma} n_\mu \left[ (2R^{\alpha\beta} - \frac{3}{4}RG^{\alpha\beta}) \delta\Gamma_{\alpha\beta}^\mu \right] &= \frac{1}{2} \int_{\partial R} d^2x \left[ -\eta^{ij}\tilde{R}(h) - 2\eta^{ij}\operatorname{tr}(h') + 4\rho h''^{ij} \right. \\
&\quad \left. + \frac{1}{2\rho}h^{ij} h_{ij} + [\rho\eta^{ij}\tilde{R}(h) - \frac{1}{2}h^{ij} \right. \\
&\quad \left. + 2\rho\eta^{ij}\operatorname{tr}(h') - 4\rho^2 h''^{ij}] h'_{ij} \right] \tag{D.2}
\end{aligned}$$

$$\int_{\partial R} d^2x \sqrt{-\gamma} n_\mu \left[ (2R^{\alpha\mu} - \frac{3}{4}RG^{\alpha\mu}) \delta\Gamma_{\alpha\beta}^\beta \right] = \frac{1}{2} \int_{\partial R} d^2x \left[ -\frac{1}{\rho}h^{ij} h_{ij} + [4\rho^2\eta^{ij} \operatorname{tr}(h'') + h^{ij}] h'_{ij} \right]$$

$$-\rho\eta^{in}\eta^{jm}[\partial^k h'_{kn} - \partial_n tr(h')]\partial_i h_{mj}, \quad (D.3)$$

$$\begin{aligned} \int_{\partial R} d^2x \sqrt{-\gamma} n_\beta [(\nabla_\mu R^{\alpha\beta} G^{\mu\nu}) \delta G_{\alpha\nu}] &= \frac{1}{2} \int_{\partial R} d^2x \left[ -\eta^{ij} \tilde{R}(h) - \rho\eta^{im} \partial^j (\partial^n h'_{nm} - \partial_m tr(h')) \right. \\ &\quad \left. - 2\rho\eta^{ij} tr(h'') - \eta^{ij} tr(h') + 2\rho h''^{ij} \right] h_{ij}, \end{aligned} \quad (D.4)$$

$$\begin{aligned} \int_{\partial R} d^2x \sqrt{-\gamma} n_\beta [(\nabla_\mu R^{\alpha\nu} G^{\mu\beta}) \delta G_{\alpha\nu}] &= \frac{1}{2} \int_{\partial R} d^2x \left[ -\eta^{ij} \tilde{R}(h) - 2\eta^{ij} tr(h') + 8\rho h''^{ij} \right. \\ &\quad \left. - \rho\eta^{ij} \tilde{R}(h') - 2\rho\eta^{ij} tr(h'') + 4\rho^2 h'''^{ij} \right] h_{ij}. \end{aligned} \quad (D.5)$$

Altogether we find

$$\begin{aligned} S_{(2)} &= \frac{1}{32\pi G} \int_{\partial R} d^2x \left[ \left(\frac{3}{\rho}\right) h^{ij} h_{ij} + (-3) h^{ij} h'_{ij} \right] \\ &\quad - \frac{1}{m^2} \frac{1}{32\pi G} \int_{\partial R} d^2x \left[ \left(\frac{3}{2\rho}\right) h^{ij} h_{ij} + \left(\frac{-3}{2}\right) h^{ij} h'_{ij} + 8\rho h'''^{ij} h_{ij} + 4\rho^2 h'''^{ij} h_{ij} \right. \\ &\quad - \eta^{ij} \tilde{R}(h) h_{ij} + 2\rho\eta^{ij} tr(h'') h_{ij} - \rho\eta^{ij} \tilde{R}(h') h_{ij} - 2\eta^{ij} tr(h') h_{ij} \\ &\quad - 4\rho^2 \eta^{ij} tr(h'') h'_{ij} + \rho\eta^{ij} \tilde{R}(h) h'_{ij} + 2\rho\eta^{ij} tr(h') h'_{ij} - 4\rho^2 h'''^{ij} h'_{ij} \\ &\quad \left. + 2\rho\eta^{im} \partial^j [\partial^n h'_{mn} - \partial_m tr(h')] h_{ij} + \rho\eta^{in} \eta^{jm} [\partial^k h'_{kn} - \partial_n tr(h')] \partial_i h_{mj} \right]. \end{aligned} \quad (D.6)$$

Setting  $m^2 = 1/2$  and using the results of the section (3.2) one find the quadratic on-shell action (4.1).

Note that to write the above expression for arbitrary  $m$  we had to consider the contribution of the Gibbons-Hawking term and another counterterm. To do this we need the form of the Gibbons-Hawking term in the Fefferman-Graham coordinates which is

$$S_{(G.H)} = \int_{\partial R} d^2x \sqrt{-g} \left( \frac{1}{\rho} g_{ij} - g'_{ij} \right) g^{ij}. \quad (D.7)$$

Therefore the variation of the Gibbons-Hawking term reads

$$\delta I_{(G.H)} = \int_{\partial R} d^2x \sqrt{-g} \left[ \left( \frac{1}{\rho} g_{ij} - g'_{ij} + \frac{1}{2} tr(g^{-1} g') g_{ij} \right) \delta g^{ij} - (g^{ij}) \delta g'_{ij} \right]. \quad (D.8)$$

Note that to cancel an infinity arising from a term like  $(\frac{1}{\rho} h^{ij} h_{ij})$  we need another counterterm given by

$$S_{c.t.2} = \frac{1}{8\pi G} \int d^2x \sqrt{\gamma} \left( -1 + \frac{1}{4} R[\gamma] \log(\rho_0) \right). \quad (D.9)$$

The variation of this counterterm in Fefferman-Graham coordinates is

$$\delta S_{c.t.2} = -\frac{1}{16\pi G} \int d^2x \sqrt{-g} \left[ \frac{1}{\rho} g^{ij} \delta g_{ij} \right]. \quad (\text{D.10})$$

## E Notations

In this appendix we present the notations used in section 4.2. Let us define the new coordinates as follows

$$z = x - t = x^1 - x^2, \quad \bar{z} = x + t = x^1 + x^2. \quad (\text{E.1})$$

Therefore

$$\partial^1 = \frac{\partial x^1}{\partial z} \partial^z + \frac{\partial x^1}{\partial \bar{z}} \partial^{\bar{z}} = \frac{1}{2}(\partial^z + \partial^{\bar{z}}), \quad \partial^2 = \frac{\partial x^2}{\partial z} \partial^z + \frac{\partial x^2}{\partial \bar{z}} \partial^{\bar{z}} = \frac{1}{2}(-\partial^z + \partial^{\bar{z}}) \quad (\text{E.2})$$

In this notation one has

$$\eta_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \eta_z^{ij} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}. \quad (\text{E.3})$$

Using the fact that

$$\partial_1 = \eta_{11} \partial^1 \quad \partial_2 = \eta_{22} \partial^2, \quad \partial^z = \eta^{z\bar{z}} \partial_{\bar{z}} \quad \partial^{\bar{z}} = \eta^{\bar{z}z} \partial_z \quad (\text{E.4})$$

one gets

$$\partial^1 \partial_1 + \partial^2 \partial_2 = \partial^z \partial^{\bar{z}} = 4 \partial_z \partial_{\bar{z}} \quad (\text{E.5})$$

Using this notations, for example, we have

$$\begin{aligned} b_{(2)11} &= b_{(2)zz} + b_{(2)\bar{z}\bar{z}} + b_{(2)z\bar{z}} + b_{(2)\bar{z}z} \\ b_{(2)22} &= b_{(2)zz} + b_{(2)\bar{z}\bar{z}} - b_{(2)z\bar{z}} - b_{(2)\bar{z}z} \\ b_{(2)12} &= -b_{(2)zz} + b_{(2)\bar{z}\bar{z}} + b_{(2)z\bar{z}} - b_{(2)\bar{z}z} \\ b_{(2)21} &= -b_{(2)zz} + b_{(2)\bar{z}\bar{z}} - b_{(2)z\bar{z}} + b_{(2)\bar{z}z} \end{aligned} \quad (\text{E.6})$$

As an example let us recast the one point functions in terms of  $z\bar{z}$  coordinates. To proceed we note that the second order expansion of action (4.7) in these coordinates is given by

$$\begin{aligned} S_{2,tot} &= \frac{1}{16\pi G} \frac{1}{2} \int_{\partial R} dz d\bar{z} \left[ 32\rho h''_{zz} h_{\bar{z}\bar{z}} + 32\rho h''_{\bar{z}\bar{z}} h_{zz} + 16\rho^2 h'''_{zz} h_{\bar{z}\bar{z}} + 16\rho^2 h'''_{\bar{z}\bar{z}} h_{zz} \right. \\ &\quad \left. - 16\rho(\partial\bar{\partial}h'_{zz})h_{\bar{z}\bar{z}} - 16\rho(\partial\bar{\partial}h'_{\bar{z}\bar{z}})h_{zz} - 16\rho^2 h''_{zz} h'_{\bar{z}\bar{z}} - 16\rho^2 h''_{\bar{z}\bar{z}} h'_{zz} + 16\rho h'_{zz} h'_{\bar{z}\bar{z}} \right]. \end{aligned} \quad (\text{E.7})$$

Using this expression the one point function of energy-momentum tensor,  $T$ , reads<sup>12</sup>

$$\begin{aligned}\langle T_{zz} \rangle &= \frac{4\pi}{\sqrt{-g_{(0)}}} \frac{\delta S_{2,tot}}{\delta h^{zz}} = \frac{4\pi}{\sqrt{-g_{(0)}}} \left(-\frac{1}{4}\right) \frac{\delta S_{2,tot}}{\delta h_{\bar{z}\bar{z}}} \\ &= 4\pi \left(\frac{1}{\frac{1}{2}}\right) \left(\frac{1}{16\pi G \times 2}\right) \left(\frac{-1}{4}\right) [16b_{(2)zz} - 16\partial\bar{\partial}b_{(0)zz}] = \frac{1}{4G} [-4b_{(2)zz} + 4\partial\bar{\partial}b_{(0)zz}].\end{aligned}\tag{E.8}$$

On the other hand for  $\langle t_{zz} \rangle$  we first need to calculate

$$\begin{aligned}\frac{1}{\rho} \frac{\delta S_{2,tot}}{\delta h'^{zz}} &= \frac{1}{\rho} \left(\frac{-1}{4}\right) \frac{\delta S_{2,tot}}{\delta h'_{\bar{z}\bar{z}}} \\ &= \left(\frac{1}{16\pi G \times 2}\right) [-4b_{(2)zz} \log(\rho) - 4g_{(2)zz} + 4(\partial\bar{\partial}b_{(0)zz}) \log(\rho) + 4\partial\bar{\partial}g_{(0)zz} + \rho - term].\end{aligned}\tag{E.9}$$

As a result we arrive at

$$\begin{aligned}\langle t_{zz} \rangle &= \frac{4\pi}{\sqrt{-g_{(0)}}} \frac{1}{\rho} \frac{\delta S_{2,tot}}{\delta h'^{zz}} - \log(\rho) \frac{4\pi}{\sqrt{-g_{(0)}}} \frac{\delta S_{2,tot}}{\delta h^{zz}} \\ &= (4\pi) \frac{1}{\frac{1}{2}} \left(\frac{1}{16\pi G \times 2}\right) [-4g_{(2)zz} + 4\partial\bar{\partial}g_{(0)zz}] = \frac{1}{4G} [-4g_{(2)zz} + 4\partial\bar{\partial}g_{(0)zz}].\end{aligned}\tag{E.10}$$

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<sup>12</sup>Note that  $\sqrt{-g_{(0)}} = \frac{1}{2}$ .

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